Adaptation of a Strapdown Formulation for Processing Inertial Platform Data

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An estimator propagation formulation, which utilizes dynamic data (attitude and sensed acceleration information) from a gimballed inertial platform, has been developed to aid in the Shuttle postflight trajectory reconstruction process and aerodynamic coefficient determination studies. Unlike classical inertial algorithms, this formulation yields a six degree-of-freedom fully coupled state and attitude estimate. Furthermore, this inertial version is shown to be independent of initial unknown platform misalignments. Results obtained using actual Inertial Measurement Unit (IMU) Data and Aeodynamic Coefficient Identification Package (ACIP) strapdown data from Shuttle flights are presented.

Introduction

THE recent missions of the Space Shuttle *Columbia* have opened a new era in manned spaceceflight. Researchers at the NASA Langley Research Center, as well as others throughout the aerospace community, have proposed use of the Shuttle as a research vehicle for postflight aerodynamic and aerothermodynamic investigations.[1-3] The best postflight trajectory and atmospheric information are a necessary input for investigations such as the Aerodynamic Coefficient Measurement Experiment (ACME). An essential requirement for postflight trajectory reconstruction is a set of dynamic data to describe the motion of the spacecraft along its re-entry flight path.

Postflight Shuttle trajectory reconstruction investigators have the opportunity to process dynamic data from one or both of two sources: 1) a body-mounted strapdown system of accelerometers and gyros comprising the Aerodynamic Coefficient Identification Package (ACIP), and 2) gimballed inertial platform of accelerometers and gyros known as the Inertial Measurement Unit (IMU). The ACIP is capable of tracking the most rapid of vehicular dynamics, outputting body attitude rates and translational accelerations at a frequency of 170 Hz, whereas the IMU data are only available at about 1 Hz. (The IMU itself operates at a higher frequency; however, properly time-tagged homogeneous sets of velocity and angle data are available postflight at the downlist telemetry/onboard recorder rate only.) Studies have shown that the specified ACIP accuracy (1% of full scale) is a limitation for best estimated trajectory (BET) generation. Though it is still necessary to be able to process ACIP data for high-frequency aerodynamic investigations, the IMU data were deemed more appropriate for the postflight trajectory reconstruction process. This is because 1) the IMU data are inherently more accurate (Table 1); 2) the 1-Hz input availability was shown to be sufficient for trajectory determinations; and 3) the mechanism by which the IMU data are accumulated makes it significantly less sensitive to data dropouts. In addition, the IMU is a tri-redundant system, allowing for unit-to-unit comparisons to aid in the selection of the best available input data.

This paper will discuss a method capable of processing inertial platform data as equivalent strapdown data. Unlike classical inertial algorithms, where the position and velocity can be estimated without estimating the vehicle attitude, this hybrid algorithm yields a six degree-of-freedom fully coupled state and attitude estimate. Because attitude must be estimated, the derived inertial formulation will be shown to be independent of initial unknown platform misalignments. Since the dynamic data are converted to equivalent strapdown data, angular rates and linear accelerations in body coordinates are directly available.

Description of the Strapdown Formulation of the Estimator

The strapdown formulation of the estimator expects as dynamic input data the measured attitude rates P_{BM}, Q_{BM}, and R_{BM} expressed about the body X, Y, and Z, axes, respectively, as well as the measured translational body accelerations a_{xBM}, a_{yBM}, a_{zBM} (see Fig. 1). The measurements may be corrected for misalignment, scale factor, and bias errors:

$$\begin{bmatrix} P_B \\ Q_B \\ R_B \end{bmatrix} = T_g \begin{bmatrix} P_{B_M} \\ Q_{B_M} \\ R_{B_M} \end{bmatrix} + \begin{bmatrix} b \\ i \\ a \\ s \end{bmatrix} + \cdots \tag{1}$$

$$\begin{bmatrix} a_{x_B} \\ a_{y_B} \\ a_{z_B} \end{bmatrix} = T_a \begin{bmatrix} a_{x_{B_M}} \\ a_{y_{B_M}} \\ a_{z_{B_M}} \end{bmatrix} + \begin{bmatrix} b \\ i \\ a \\ s \end{bmatrix} + \cdots \tag{2}$$

where T_g and T_a are 3×3 transformation matrices that correct the measurements for misalignment and scale factor errors, and the elements of the T matrices and bias vectors are

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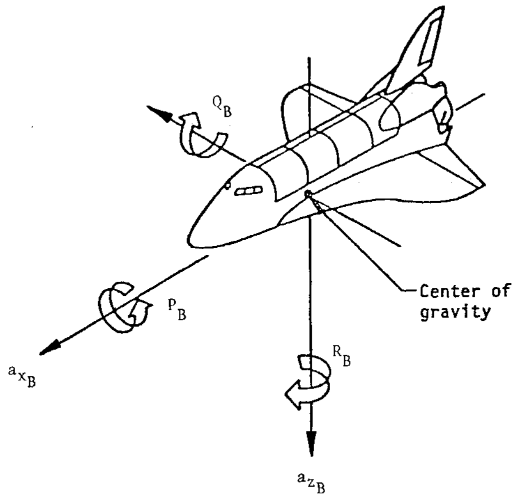

Fig. 1 Definition of body axis acceleration and attitude rates.

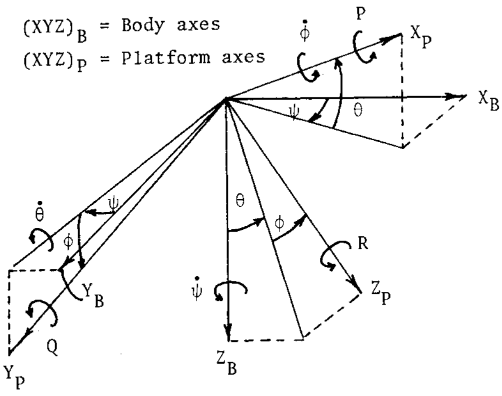

Fig. 2 Euler angle and attitude rate definitions.

assumed to be known a priori or solved for in an extended solution set. Note that the error terms are expressed in the sensor (body-mounted) coordinates, and hence, can be assumed to be nearly constant. Additional error terms, such as g-sensitive gyro drifts, accelerometer scale factor asymmetry or nonlinearity, etc., can be modeled if desired or required.

The corrected PQR rates are then integrated to yield an attitude time history consisting of Euler angles (or quaternions) which relate the body attitude to the inertial frame of reference. Using this information, the corrected body accelerations can be rotated into the position and velocity inertial frame of reference for state integration.

Adaptation for Inertial Platform Input

The output of the Shuttle IMU consists of accumulated sensed velocities (expressed in inertial mean of 50 coordinates)¶ and measured platform-to-body quaternions, which are derived from the gyro resolver angle readings. In order to be compatible with a strapdown integrator, these data must be converted from quaternions and velocities to angular rates and accelerations. This is done in a software preprocessor where quaternion-derived Euler angles and the accumulated velocities are spline fitted, differentiated, and output at a desired rate. The Euler angle rates can be used directly to obtain P_{P_M}, Q_{P_M}, and R_{P_M}, which represent the inertial rotation rate of the body relative to the actual (drifting) platform expressed about the actual (misaligned) platform axes coordinate system as follows:

$$
\begin{aligned}
P &= \dot{\phi} - \dot{\psi}\sin\theta \\
Q &= \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta \\
R &= \dot{\psi}\cos\phi\cos\theta - \dot{\theta}\sin\phi
\end{aligned} \tag{3}
$$

where the Euler angles ψ, θ, and ϕ and Euler angle rates $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ are as defined in Fig. 2. The measured accelerations, obtained from the spline-differentiated velocities, are rotated from the M50 coordinate system to the actual (misaligned) platform system using the (constant) M50 to platform transformation matrix yielding a_{xP_M}, a_{yP_M}, a_{zP_M}. At this point, a pseudo strapdown formulation of dynamic data output is available for processing, albeit measured with respect to the platform axes rather than the body axes.

Using the existing strapdown formulation, the "measured" platform accelerations and angular rates can be corrected for bias and scale factors errors, which are assumed to be known a priori or solved for in an extended solution set:

$$
\begin{bmatrix} P_P \\ Q_P \\ R_P \end{bmatrix} = \begin{bmatrix} P_{P_M} \\ Q_{P_M} \\ R_{P_M} \end{bmatrix} + \begin{bmatrix} b \\ i \\ a \\ s \end{bmatrix} \tag{4}
$$

and

$$
\begin{bmatrix} a_{xP} \\ a_{yP} \\ a_{zP} \end{bmatrix} = \begin{bmatrix} (1+k_x) & 0 & 0 \\ 0 & (1+k_y) & 0 \\ 0 & 0 & (1+k_z) \end{bmatrix} \times \begin{bmatrix} a_{xP_M} \\ a_{yP_M} \\ a_{zP_M} \end{bmatrix} + \begin{bmatrix} b \\ i \\ a \\ s \end{bmatrix} \tag{5}
$$

where k_x, k_y, and k_z are the x, y, and z platform accelerometer scale factor errors, respectively. Notice that the error terms are expressed in the sensor (platform) coordinates, and as such, can be expected to be nearly constant. Also, there is no such error term as a scale factor applying to IMU-derived angular rates since the IMU gyros measure whole angles (and not angular rates). Finally, observe that the "corrected" PQR rates and a_{xP}, a_{yP}, and a_{zP} accelerations are still expressed with respect to an initially misaligned platform.

Using the quaternion determined Euler angles, the "corrected" PQR rates and translational accelerations can be rotated directly from the misaligned platform coordinate system to the body coordinate system, yielding a truly equivalent strapdown dynamic data output in body coordinates. From there on, the integration of the equations of motion, the processing and filtering of external tracking data, etc., is identical.

Results

The previously described modifications were made to the existing estimation program ENTREE,[5] whose equations of motion are formulated in a strapdown coordinate system. IMU data obtained from re-entry flights of the Space Shuttle *Columbia* were then processed as the dynamic data source. The resulting best estimated trajectory (BET) provided an accurate time history of position, velocity, and attitude of the Shuttle.[6,7]

Figures 3a-f are plots of the dynamic data throughout the final 40 min of a Shuttle re-entry; in this case, the STS-1 flight on April 14, 1981. Shown thereon are the time histories of the

¶An inertial coordinate system defined by the mean Earth equator orientation (no nutation) on Jan. 1, 1950.

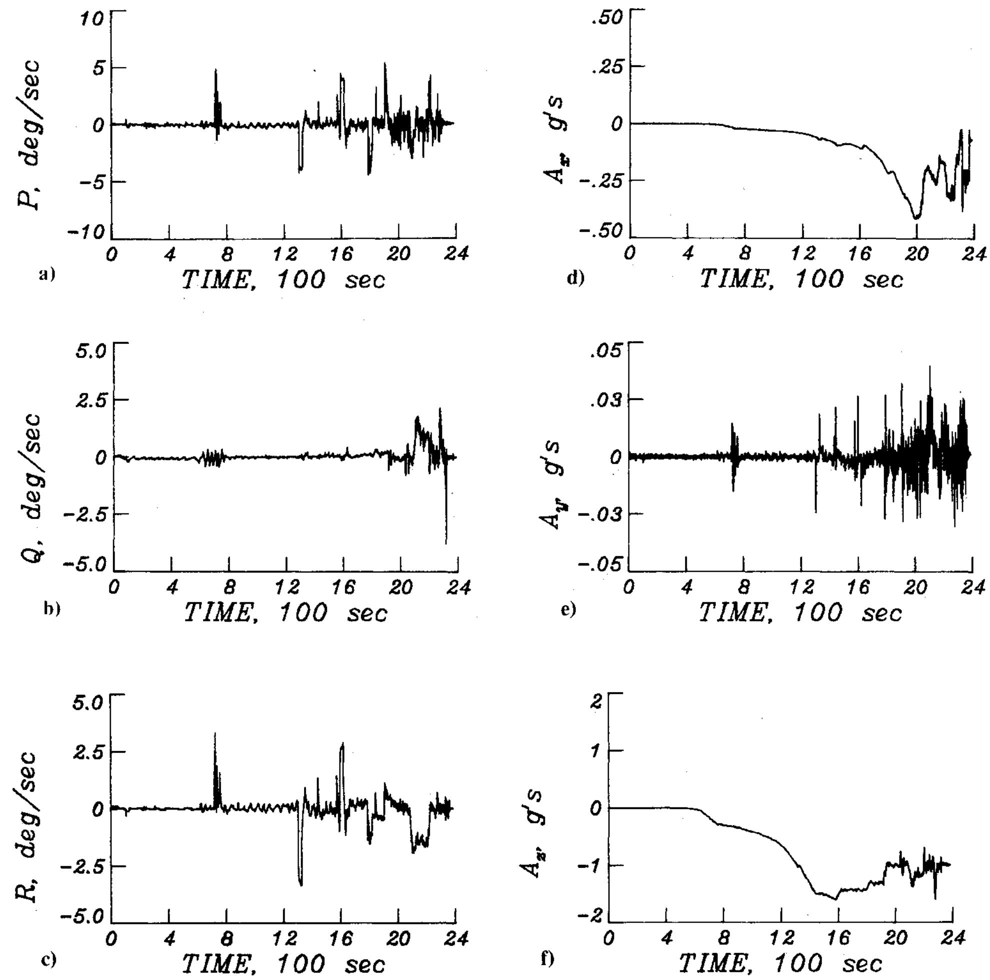

Fig. 3 Plots of dynamic data for final 40 min of STS-1: a) Roll rate with time; b) Pitch rate with time; c) Yaw rate with time; d) *X*-body acceleration with time; e) *Y*-body acceleration with time; f) *Z*-body acceleration with time.

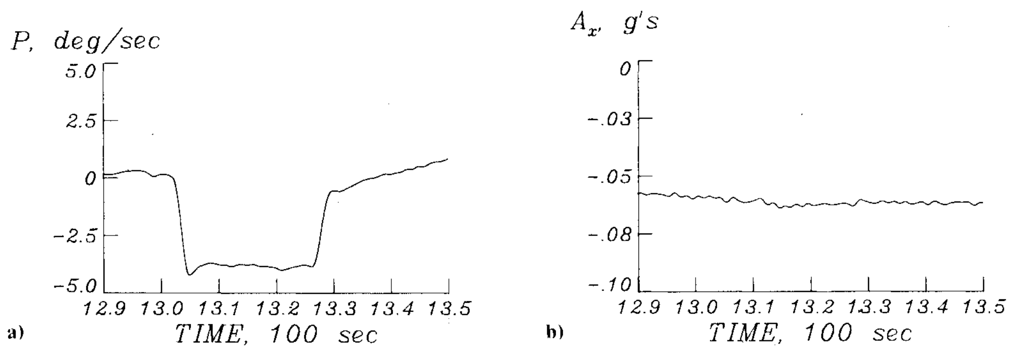

Fig. 4 IMU derived components of dynamic data during STS-1 roll reversal: a) Roll rate with time; b) *X*-body acceleration with time.

roll, pitch, and yaw angular rates expressed in degrees per second, and body axis linear accelerations expressed in *g*'s. These data were generated from the output of IMU #2. The first roll maneuver can be seen starting at $t = 710$ s where time is referenced with respect to an epoch of 63750 s past midnight, April 14, 1981. Subsequent bank reversals occur at 1300, 1590, 1780, and 1895 s. The vehicle entered into the heading alignment circle phase at about $t = 2090$ s. Touchdown occurred at $t = 2307$ s, with the vehicle coming to rest at $t = 2367$ s.

Figures 4a-b represent a closer look at the roll rate *P*, and *X*-acceleration, a_x, over a 60-s period spanning the first roll reversal. Figures 5a and 5b show the same data as output by the ACIP. Recalling that the ACIP outputs body rates and linear accelerations directly in body coordinates, a comparison of Figs. 4 with Figs. 5 demonstrates that the inertial IMU data are being correctly derived and converted into the proper (body) components. Note also that the "thinned," 50-Hz ACIP data are much noisier (reflecting spacecraft structural modes) than the smooth, spline-fitted 1-Hz IMU data. Figures 6a and 6b show the difference between the ACIP and the IMU over the same time period. Compared to the more accurate IMU, biases can be seen in the 1% specified accuracy ACIP data. The mean difference, μ, and standard deviation, σ, are printed in the upper right corner of each plot.

Advantages of Using the IMU for Dynamic Data Generation

As alluded to earlier, the inherent accuracy of the IMU makes it desirable as a source of dynamic data for postflight trajectory determination. Systematic errors, such as gyro drift, accelerometer bias and scale factor, *g*-sensitive terms, etc., can be modeled in an attempt to simulate the actual

Table 1 Major IMU Error source specifications during re-entry[4]

Component	Error source	Specified accuracy (1σ)
Accelerometer	Bias	50μg (uncalibrated) 10 μg (calibrated-approximate)
	Scale factor	100 ppm
	Quantization	1 cm/s
	Noise	5 μg
Gyro	Bias drift	0.035 deg/h (uncalibated) 0.022 deg/h (calibrated-approximate)
	g-Sensitive drift	0.025 deg/h/g
	g^2-Sensitive drift	0.025 deg/h/g^2
	Resolver quantization	20 arcsec
	Resolver noise	12 arcsec

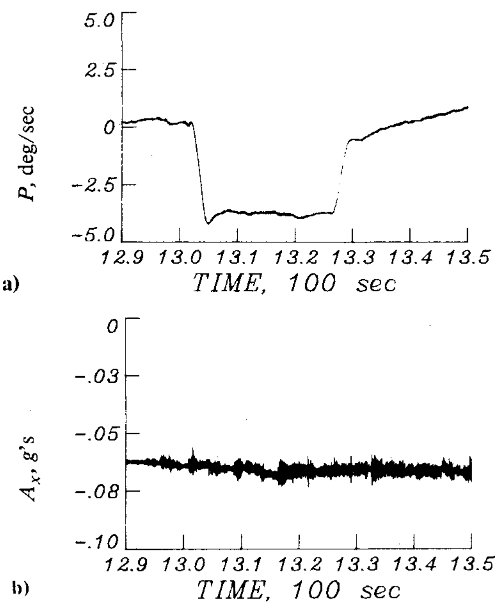

Fig. 5 ACIP derived components of dynamic data during STS-1 roll reversal: a) Roll rate with time; b) X-body acceleration with time.

hardware performance. The presence of these terms in the estimation algorithm generally contributes to a trajectory solution which more closely fits the measurement data. However, the accuracy with which these terms may be solved for is dependent on the accuracy of the tracking data itself and the presence of any other unmodeled systematic or nonsystematic errors.

Examples of nonsystematic errors in the IMU include instrument noise and quantization of both the gyro resolver and linear accelerometer output. For instance, accumulated velocity is quantized to multiples of 1 cm/s over an accumulation interval. At an approximate 1-Hz data availability rate, this translates into *locally* derived maximum acceleration errors of up to 1 cm/s/s ≈ 0.033 ft/s^2 ≈ 1.0 mg. Likewise, in the attitude channel, a 20 s gimbal resolver quantization coupled with the 12 s gimbal resolver noise can map into ≈ 0.01 deg/s local attitude rate error. However, since the hardware accumulators are of a *nondestruct* design, these quantization induced locally derived rate errors are nonpropagating, in the sense that said errors at time t plus Δt are

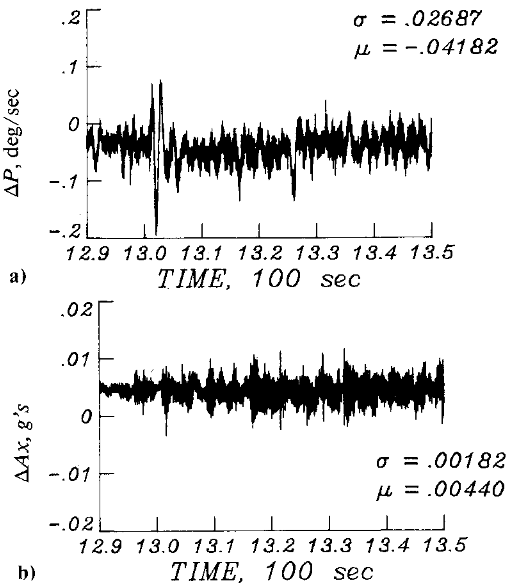

Fig. 6 IMU minus ACIP dynamic data differences during STS-1 roll reversal: a) Roll rate differences with time; b) X-body acceleration differences with time.

unaffected by errors at time t. Simulation has shown that over the time interval of a Shuttle re-entry, the effects of the quantization average out to a zero mean in the dynamic data itself, and contribute only a modest amount to state prediction errors. The effect of quantization on dynamic data determination can be seen in Fig. 7a and 7b. Plotted thereon are 30 s of yaw rate, R, and X-acceleration, a_x, obtained from Fig. 3. This is a high altitude quiescent region of the flight with no significant ongoing rates or thrusting ($t=10$ s to $t=40$ s). The local 1-mg oscillation in linear acceleration and 0.01 deg/s error in attitude rate can readily be seen. Although evident during this quiescent mode, the effects are nearly invisible when superimposed with actual ongoing accelerations, as was shown in Fig. 3.

Since the IMU determined body rates and linear accelerations are derived from whole gimbal angles and accumulated velocities (and since the instrument is of a nondestruct nature, as has been pointed out), the effects of gaps in the dynamic data are minimal. This is because an interpolator can create rates and accelerations which, when integrated, yield exactly the angles or velocities at the endpoints of the gap. [This is in contrast to an instrument which outputs rates (accelerations) directly; here, the missing data must be guessed—the resulting rates (accelerations) may not integrate exactly into the attitude (velocity) time history.] For all intents and purposes, the dynamic data errors induced by the data gaps are nonpropagating and are constrained to the region in the vicinity of the gap. This is illustrated in Figs. 8a and 8b, which span the same 60-s period during the first STS-1 roll reversal. A conservatively large 7-s input data gap was introduced at $t=1305$ s, midway through the dynamics of the first roll reversal maneuver. Figure 8 shows the differences between the dynamic data derived from the input data with the gap, and the gap free dynamic data previously shown in Fig. 4. As can be seen, the error due to the artificially created data gap was pretty well contained to the region spanning the occurrence of the gap, from about 1301 to 1316 s.

As an aside, though not related to the advantage of using the IMU data per se, the method of interpolation chosen—cubic spline—not only passes through each input data point exactly, therefore guaranteeing that the integration of the rates and accelerations will exactly recapture the input angles and velocities, but does so with a continuous first and second derivative. The relatively sparse IMU input data availability precluded any need to investigate the use of weighted smoothing splines.

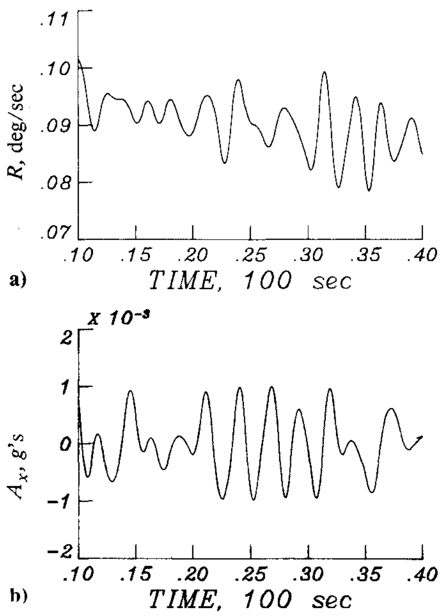

Fig. 7 IMU derived components of dynamic data from STS-1 showing effects of quantization: a) Yaw rate with time; b) *X*-body acceleration with time.

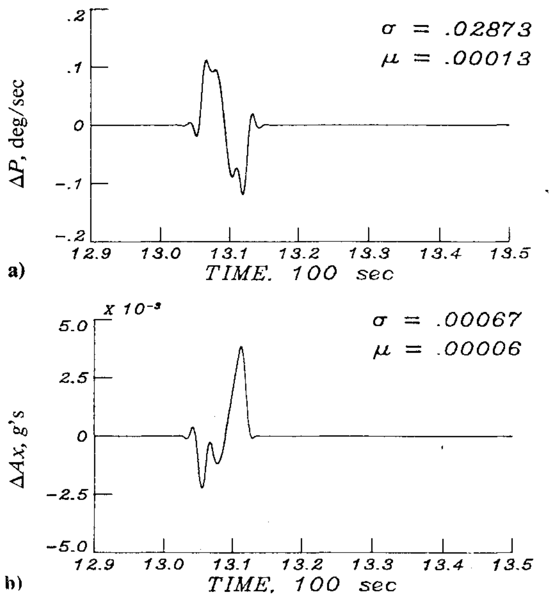

Fig. 8 Dynamic data differences resulting from a 7-s input data gap from $t = 1305$ s to $t = 1312$ s: a) Roll rate differences with time; b) *X*-body acceleration differences with time.

Comparison of the Hybrid Inertial Formulation With a Classical Inertial Formulation

A re-examination of the hybrid inertial derivation reveals that the formulation is *independent* of any platform misalignments which might be present, whether modeled or not.** This is because the required spacecraft attitude rates and accelerations are measured with respect to the actual platform axes. Whether or not the platform is oriented correctly with respect to an inertial coordinate frame defined by the stars is irrelevant. The only quantities required for the hybrid formulation are the rates of the body with respect to the IMU platform axes (whether misaligned or not). The hybrid formulation does require an estimate of body attitude simultaneous with position and velocity in order to map the body accelerations into the coordinate frame of the position and velocity estimate. This assures a synchronous coupled state required by investigators for aerodynamic and aerothermodynamic studies. This coupling of attitude and state results in an attitude estimate determined from the processing of external observations.

In a standard inertial estimator propagation algorithm, platform misalignment must be solved for in order to correctly map the measured accelerations into the inertial frame of reference and obtain comparable propagation accuracies. On the other hand, the attitude is decoupled from the position and velocity and can be deterministically derived from the IMU gimbal angles. Thus, the attitude determined in this manner is directly proportional to any errors in the estimate of the platform alignment.

The treatment of inertial platform data as equivalent strapdown data allows for a body axis attitude rate and linear acceleration determination independent of the construction of a best estimated trajectory. This is an advantage to aerodynamic investigators who have a need to quickly obtain access to the dynamic data for purposes of their experiment.

**The initial attitude estimate, if obtained from IMU data, would be in error by an amount equal to the misalignment.

Conclusions

Results have been presented using actual Shuttle IMU and ACIP data which demonstrate that the Inertial Measurement Unit output have been successfully transformed to body axis accelerations and angular rates. The advantages of this adaptation of a strapdown formulation of the equations of motion for the processing of inertially measured dynamic data are threefold: 1) The propagation algorithm is able to utilize the inherent advantages of the IMU data, while 2) the advantages of a strapdown formulation are maintained. In addition, 3) the modified formulation is insensitive to any IMU initial platform alignment errors that might be present. Inherent IMU advantages include increased accuracy, triredundant selection availability, and insensitivity to data gaps. Inherent advantages of the strapdown propagation formulation include a direct input of body axis attitude rates and linear accelerations (which are required for use by aerodynamic investigators), and a fully coupled state and attitude, allowing for direct estimation of vehicle attitude using external tracking measurement processing.

Acknowledgment

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